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Stability Characteristics of Laminated Cylindrical Panels Under Transverse Loading

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Static and dynamic approaches for the stability analysis of a laminated composite arch, a laminated composite cylindrical panel, and an elastic isotropic cylindrical panel subjected to transverse loading are discussed. The inspiration of this stability analysis during the snapping process is based on the phenomenon of energy transfer from potential energy to kinetic energy as the applied load reaches the critical value. Some important features in dynamic analysis characterizing the snapping motion, which are not observed in static analysis, are shown. Different snapping phenomena in dynamic analyses are presented in terms of variations in depth and effects of anisotropy.

Introduction

THE stability analysis of laminated panels under transverse loading merits a great deal of attention due to its occurrence in practical applications. Thin laminated panels under transverse loads could encounter deflections of the order of the shell thickness or greater. Laminated cylindrical panels subjected to transverse static loading could also result in the snapping through phenomenon, which characterizes an instantaneous dynamic instability. Some researchers applied static analysis for such instantaneous instability problems, whereas others argue that the dynamic analysis should be employed to cope with the complex transient phenomenon the snapping process. Nevertheless, both static and dynamic approaches dealing with the stability problem in the occurrence of snapping and/or buckling have been studied in the past.[1] The nature of the stability for elastic media was discussed[2] thoroughly in terms of the energy criterion. Static and dynamic buckling behaviors of elastic structures were investigated,[3] in which definitions of dynamic buckling loads, estimates of the buckling loads, and assessments of the accuracy of these estimates were presented. The objective of this research is to investigate the fundamental concept and basic issues of instantaneous instability for geometrical nonlinear laminated panels when snapping occurs and to identify some important features in both static and dynamic analyses.

Because of the high ratio of in-plane elastic modulus to transverse shear modulus in laminated composite structures, transverse shear deformation plays a much more important role in reducing the effective flexural stiffness of laminated plates and shells.[4] Snapping stability response of these kinds cannot be predicted correctly by using the small or intermediate displacement theory. In addition, the transverse shear deformation, which is usually neglected in the classical shell theory, cannot be ignored in the stability analysis of laminated composite shell structures subjected to transverse loading. As a result, the need to include large displacement, large rotation theory in studying the stability characteristics of thin laminated panels is of great importance. In the present study, the kinematics of large displacements and rotations for small strain situations are developed based on the total Lagrangian theory. Additionally, the transverse shear strains are assumed to distribute parabolically across the thickness and vanish at the top and bottom surfaces.

A developed 36-degree-of-freedom, curved, quadrilateral, thin shell element, for large displacement/rotation nonlinear vibration considerations of a cylindrical panel, is employed. The Riks[5] nonlinear solution technique is invoked for the load-deflection curve in the static analysis. Moreover, algorithms based on the Newton-Raphson iterative method and the beta-m[6] method, which is a generalization of the Newmark time-marching integration scheme, are incorporated in the dynamics solution.

Static and Dynamic Analysis

The static calculations result in nonuniqueness of solutions considering the nonlinear characteristics since multiple solution paths may exist. A dynamic analysis in a geometrical nonlinear problem would avoid numerical difficulties and lead to a physically unique solution as long as no material nonlinearity is considered.

The nature of snapping can be characterized as the energy transfer from the potential energy to kinetic energy due to the instability. The static solution is found through the variational principle by minimizing the potential energy for a given load. Therefore, it is possible to develop the load-displacement curve through a variation of the potential energy function of an equilibrium state variable. Each point on the curve represents another state of equilibrium. On the other hand, from the dynamic point of view, the initial conditions introduce the total energy, which is the sum of potential energy and kinetic energy, of the system. The kinetic energy makes the system move harmonically about the point presenting the minimum potential energy, i.e., the static solution, within an energy well in the energy profile of the system. When the snapping occurs, the corresponding energy well is no longer valid to give a

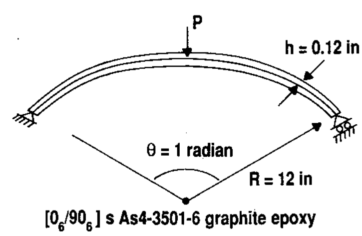

Fig. 1 Geometric configuration of a simply supported laminated arch with 24 plies and $[0_6/90_6]_s$ stacking sequence.

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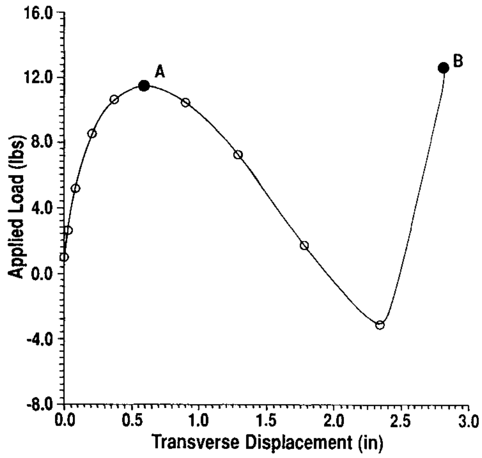

Fig. 2 Static nonlinear load-deflection curve for the point where the load is applied to the arch shown in Fig. 1.

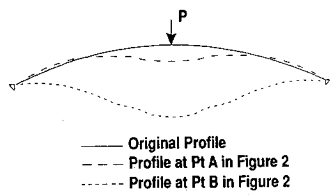

Fig. 3 Undeformed and deformed profiles of the arch shown in Fig. 1.

minimum point of potential energy. Thus, the jump of potential energy provides kinetic energy and results in the instantaneous dynamic motion. A one-degree-of-freedom model along with the energy curve has been shown and discussed thoroughly.[7] A two-degree-of-freedom model with the energy surface is also presented in the literature.[1,7] As can be seen, this energy transfer due to the instability motivates the present research to investigate the snapping and/or/buckling process from the dynamic consideration.

Work pertaining to the snapping response due to a static load has been presented.[8] In that study, the static nonlinear load-deflection relations were shown for different effects of anisotropy, fiber orientation of cylindrical laminated panels. Note that each load-displacement curve contains a stable state and an unstable state. Stability is only a matter of passing the unstable region when the load reaches the critical value. It can be shown from a dynamic point of view that each stable state can be represented by a solution that can be predicted through a displacement-time function in which a steady-state condition exists (incorporating the material damping). The study incorporated herein traces the static conditions of equilibrium by first evaluating the nonlinear equations of motion, without damping, for various step load-time functions.

$$[M]\{\ddot{u}\} + [K]\{u\} = \{P(t)\} \tag{1}$$

where $\{u\}$ is the displacement vector, $\{P(t)\}$ the time-dependent applied load, $[M]$ the consistent mass matrix, and $[K]$ the nonlinear stiffness matrix, which can be written as

$$[K] = \left[K + \frac{N_1}{2} + \frac{N_2}{3}\right] \tag{2}$$

where $[K]$ is a constant stiffness matrix, $[N_1]$ a linear stiffness matrix, and $[N_2]$ a quadratic stiffness matrix. The solutions to two load levels are traced. For a load below the collapse (therefore in the stable region), the displacement vs time curve almost immediately oscillates about a displacement value equivalent to the static or steady-state solution. If a load value is just above the collapse, or jump point, snapping occurs. This phenomenon can be traced using a dynamic solution technique, e.g., beta-m method. A much more complicated oscillating curve is observed when such a snapping situation is involved. A laminated composite arch is first studied. This study is then extended to a laminated composite cylindrical panel. These two structures are considered to be relatively deep. Next, the nonlinear dynamics response of an elastic isotropic cylindrical shallow panel[9] is introduced for comparison. This last reference gives a complete development of the dynamic finite element equations. The various depths among those cylindrical panels considered explains the complicacy of the oscillating curve that will be seen in the illustration problems. Furthermore, two load levels that are lower than the critical snapping load are incorporated in the elastic isotropic cylindrical problem to present the solutions during the snapping motion. Results, as shown in the illustration problem, indicate that the last two load levels yield a dynamic stable response with steady-state displacements similar to those observed in the static load-displacement curve. The validity of using the motion in a single degree of freedom direction of one discrete point to simulate the actual two-dimensional motion of the shell surface is also discussed.

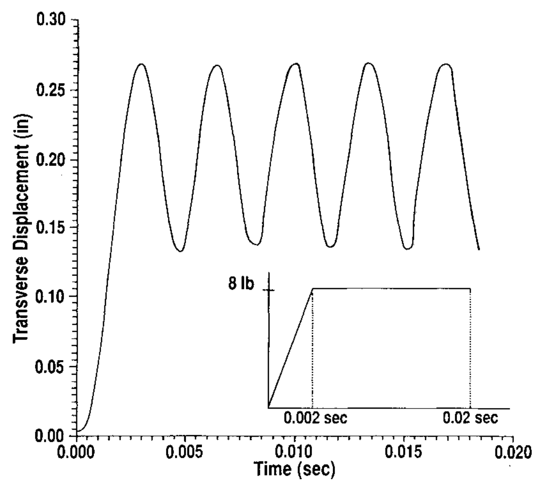

Fig. 4 Loading history and dynamic response vs time for the center of the arch shown in Fig. 1 when the applied load is lower than the critical snapping load.

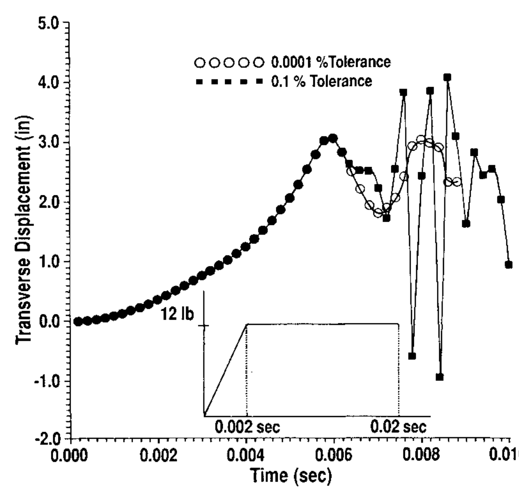

Fig. 5 Loading history and dynamic response vs time for the center of the arch shown in Fig. 1 when the applied load is slightly greater than the critical snapping load.

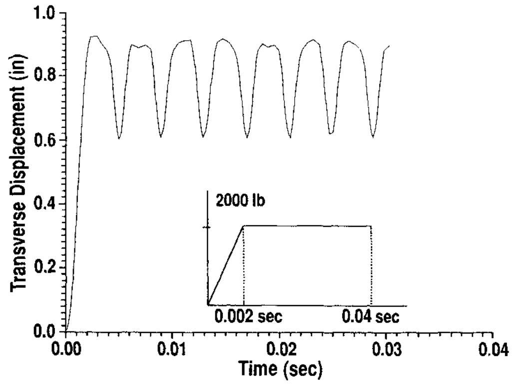

Fig. 6 Dynamic response for the center of the laminated composite cylindrical panel when the applied load is lower than the critical snapping load.

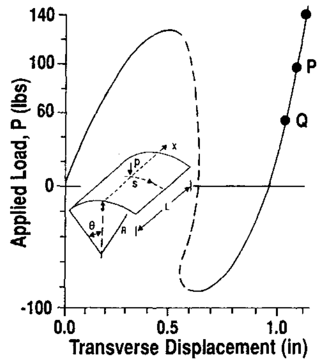

Fig. 8 Geometric configuration of a simply supported elastic isotropic cylindrical panel and the static nonlinear load-deflection curve for the point where the load is applied.

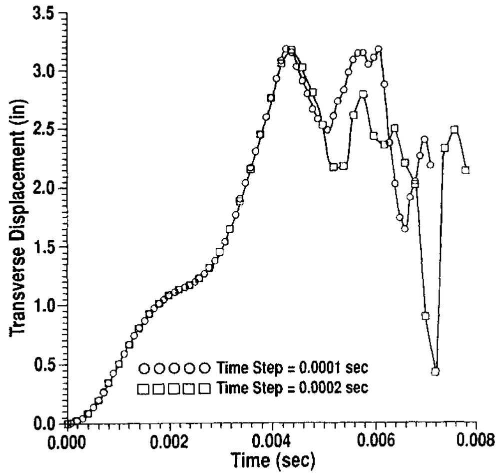

Fig. 7 Dynamic response for the center of the laminated composite cylindrical panel when the applied load is slightly beyond the critical snapping load with two different time steps: 0.0001 and 0.0002 s.

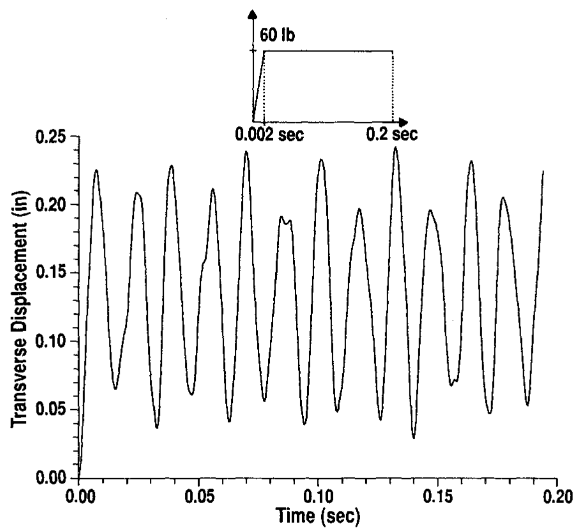

Fig. 9 Loading history and dynamic response for the center of the elastic isotropic cylindrical panel when the applied load is lower than the critical snapping load.

Mathematic Formulations of Dynamic Solution

The beta-m[6] method, which is a generalization of the Newmark time-marching integration scheme, along with the Newton-Raphson iterative technique, is used in the present research for the dynamics analysis and will be presented for completeness. This method provides a general single-step algorithm applicable to initial value problems and is specialized by specifying the method order m along with m integration parameters, $\beta_0, \beta_1, \beta_2, \ldots, \beta_{m-1}$. For a particular choice of m, the integration parameters provide a subfamily of methods that control accuracy and numerical stability. The beta-m method is defined as

$$u_{n+1}^{(k)} = q_k + b_k \Delta u^{(m)} \tag{3}$$

where

$$q_k = \sum_{j=k}^{m} \frac{u_n^{(j)} h^{j-k}}{(j-k)!} \tag{4}$$

$$b_k = \beta_k h^{m-k}/(m-k)! \tag{5}$$

and $k = 0,1,2,\ldots,m$. The β_m in Eq. (5) is defined to be equal to 1, and h is the time increment for the time step n. The method order m implies that $u_n^{(m)}$ is the highest derivative to be retained. In the present study, $m = 2$, which happens to be in the case of the Newmark time-integration scheme. Additionally, $\beta_0 = 0.5$, $\beta_1 = 0.5$, and $\beta_2 = 1.0$, are suggested[6] for an unconditionally stable analysis.

If one substitutes Eq. (3) into Eq. (1) at time t_{n+1}, the result is

$$[b_2 M + b_0 K(q_0 + \Delta u^{(m)})]\Delta u^{(m)} = P_{n+1} - \{Mq_2 + K(q_0 + b_0 \Delta u^{(m)})q_0\} \tag{6}$$

where P_{n+1} is the applied load at time t_{n+1}; b_0, b_1, and b_2 are scalars dependent on the integration parameters, as shown in Eq. (4). Equation (6) results in a set of nonlinear algebraic equations. Thus, the Newton-Raphson iterative method is incorporated herein to solve Eq. (6). At each time step, the following assumption is made:

$$\Delta u_{i+1}^{(m)} = \Delta u_i^{(m)} + \delta u_i^{(m)} \tag{7}$$

where i is the iteration number. Applying the Newton-Raphson method to Eq. (6) and using Eq. (7) gives

$$[b_2 M + b_0 K_T(q_0 + b_0 \Delta u_i^{(m)})]\delta u_i^{(m)} = P_{n+1} - M\{q_2 + b_2 \Delta u_i^{(m)}\} - K(q_0 + b_0 \Delta u_i^{(m)})\{q_0 + b_0 \Delta u_i^{(m)}\} \tag{8}$$

where K_T is the updated tangential stiffness matrix and is written as

$$K_T = [K + N_1 + N_2] \tag{9}$$

As a result, Eq. (8) gives the basic algorithm for the dynamic analysis in the present research. It should be noted that the solution of the equation is a by-product of the tolerance incorporated in the Newton-Raphson scheme as well as the time-step value, which is a characteristic of the given problem.

Examples

A 24-ply $[0_6/90_6]_s$ simply supported laminated arch made from Hercules AS4-3501-6 graphite expoxy subjected to a transverse concentrated load applied at the center of the arch is employed for the first illustration problem. The arch has a thickness $h = 0.12$ in., a width in the same order of thickness at 0.12 in., a radius of curvature $R = 12$ in., along with an open angle $\theta = 1.0$ rad., as shown in Fig. 1. Eight elements with size ranging from 1×0.06 to 0.25×0.06 in. are used to simulate a quarter of the arch due to its symmetric condition. The static nonlinear load-deflection curve for the point where the load is applied is shown in Fig. 2, where point A indicates that the critical snapping load is approximately 11.5 lb and point B is the snapping through point.

Figure 3 displays the underformed and deformed profiles of the arch. The arch snaps instantaneously from profile A to profile B and is unstable until reaching profile B, where it is able to resume sustaining a load. The movement to the profiles shown in Fig. 3 corresponds to the instantaneous unstable movement from point A to point B in Fig. 2. Furthermore, the dynamic analysis is incorporated to investigate the instantaneous instability during the snapping process. A step load, with the load amplitude of 8 lb, which is lower than the critical snapping load, is applied in the dynamic analysis. The loading history and the response vs time are shown in Fig. 4. As can be seen, the response oscillates periodically and reaches the steady-state condition. Moreover, the periodic response oscillating about the equilibrium position represents the static solution shown in Fig. 2.

As the applied load becomes greater than the critical snapping load, the dynamic response behaves in a totally different fashion. Figure 5 shows the loading history and the oscillating feature for a step load with the load amplitude of 12 lb, which is slightly greater than the critical snapping load. Additionally, the tolerance for the convergence in beta-*m* time-integration scheme plays an important role for obtaining a realistic solution. Figure 5 also shows the dynamic solutions for two different tolerances, 0.0001 and 0.1%, respectively. Clearly, the finer tolerance gives a more plausible oscillating response.

The next study considers a simply supported laminated composite cylindrical panel subjected to a transverse center concentrated load. The materials and stacking sequence are the same as the previous arch. Again, a radius of curvature $R = 12$ in. is used along with an open angle $\theta = 1.0$ rad. The sizes of 49 elements varying from 2×1 in. to 0.5 in. are employed for the analysis of a quarter of the panel due to the symmetry. A length of 18 in. is used herein. The static analysis

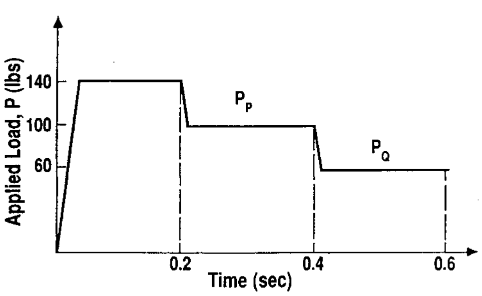

Fig. 10 Loading history used for the solution of the elastic isotropic cylindrical panel during the snapping process.

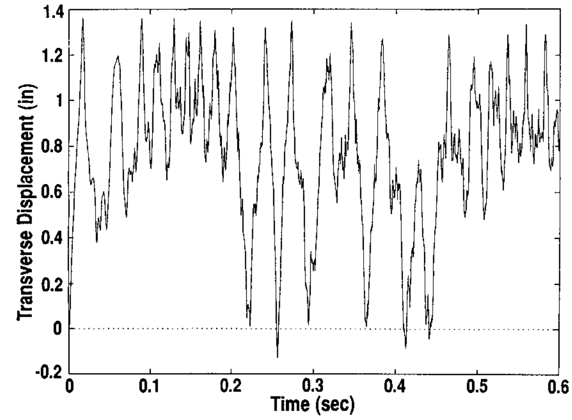

Fig. 11 Dynamic response at the center of the elastic isotropic cylindrical panel for the loading history shown in Fig. 10.

of this problem has been studied[10] and shows the critical snapping load to be approximately 2800 lb.

A similar step load is used as in the previous case with the load capacity 2000 lb employed for the dynamic analysis. Figure 6 shows the periodic fashion of the response when the applied load is lower than the critical snapping load. Similar to Fig. 4, Fig. 6 shows the steady-state condition and the corresponding static solution. However, as the applied load reaches the critical snapping load, the solution becomes erratic. Figure 7 presents the dynamic response for two different time steps when the load capacity of applied step load becomes 3000 lb, which is only slightly beyond the critical snapping load. The smaller time step of 0.0001 s obviously gives a better result than the large time step of 0.0002 s. A reasonable periodic feature is observed for only 1.5 cycles for the time step 0.0001 s, and then the solution starts to behave in a fashion that can be classified as a non-steady-state solution. It is believed that the large number of calculations performed, the nonlinearity of the strain-displacement equations, the precision in the computer code, and the roundoff of the computer may be reasons for variations in the solution. Yet, it is also possible that the function being studied, a discrete point oscillation, is no longer valid for expressing the panels' nonlinear movement. The authors next studied an elastic isotropic cylindrical panel for the dynamic analysis in order to explore the unrealistic situation in this case and to further identify the physical meaning during the snapping process.

A simply supported elastic cylindrical panel subjected to transverse concentrated load, as previously applied, is shown in Fig. 8. The elastic panel has the following material and geometric properties: Young's modulus $E = 450{,}000$ psi, Poisson ratio $\nu = 0.3$, open angle $\theta = 0.1$ rad, radius of curvature $r = 100$ in., thickness $h = 0.25$ in., and length $L = 20$ in. One should note that the elastic panel is considered a shallow one. The boundary conditions are characterized as free boundaries along $x = \pm 10$ in., and $u = v = w = \phi_1 = 0$ along $s = \pm 10$ in. The lines along $x = 0$ in. and $s = 0$ in. are lines of symmetry. A quarter of the shell is therefore modeled to simulate the entire structure due to the symmetric character. Moreover, a mesh with 4×6 elements is used to idealize the quarter of the panel. Figure 8 gives the geometric description of the elastic panel as well as the static or steady-state load-deflection curve of the point where the load is applied. As can be seen, the solid line portion delineates a stable state, whereas the dashed line portion is an unstable state. Each point along the stable portion of the curve represents a physical state of equilibrium, which can be identified as the steady-state condition in the dynamic analysis.

A typical oscillating feature in the dynamic analysis for the left part of the solid line, i.e., the stable state in a loading process where the load is lower than the critical snapping load, is shown in Fig. 9. A step load with a magnitude of 60 lb for

0.2 s is employed for the dynamic analysis using 500 time steps. Clearly, the steady-state response shown in Fig. 9 oscillates about the displacement value equivalent to the static solution in Fig. 8. When an applied load exceeds the critical snapping load, the loading history given in Fig. 10 is incorporated to obtain steady-state solutions for three load levels: the load level slightly above the snapping critical load and the load levels P_P and P_Q. The rate of load changes between any two adjacent load levels remain unchanged. The two load levels P_P and P_Q, which are lower than the critical snapping load, characterize the solutions during the snapping motion. Points P and Q in Fig. 8 correspond to load levels P_P and P_Q in Fig. 10, respectively. Figure 11 represents the dynamic response at the center of the elastic panel when the load history shown in Fig. 10 is applied. The global steady-state feature, along with the snapping characteristics at the beginning of motion, are observed in Fig. 11.

Discussion and Conclusions

Snapping of a cylindrical panel under a transverse load is clearly a dynamic phenomenon. A laminated composite deep arch, a laminated composite deep panel, and an elastic isotropic cylindrical shallow panel are used for the study of instantaneous stability during the course of snapping. Prior to the critical snapping load, the steady-state condition in dynamic analysis is reached almost immediately for these illustration problems. This steady-state solution yields the corresponding static solution in the load-deflection curve. However, when the applied load exceeds the critical snapping load, a drastic change in the oscillatory fashion is observed. The difference can be seen by comparing Figs. 5, 7, and 11. The dynamic responses of a deep arch and deep composite panel give a periodic feature after the snapping occurs, as shown in Figs. 5 and 7. On the contrary, it is difficult to identify the steady-state condition in the dynamic response for the shallow elastic panel. This difficulty is seen in the beginning part of Fig. 11. It is believed that the depth of the arch gives an obvious periodic function represented by the discrete transverse displacement vs time for a specific point. In the composite panel, this unique point function gives way to a more complex two-dimensional functional of energy. It becomes even more obvious when the shallow isotropic panel is considered.

Though this paper is directed toward laminated cylindrical panels, an illustration of the method incorporated in this study can be traced by considering a cylindrical shallow panel made from isotropic materials. It is possible to determine the phenomenon of collapse as well as the characteristics of stability for a cylindrical panel by comparing the nonlinear dynamic solution with the nonlinear static results.

Acknowledgments

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